Endogenous Information Acquisition in Coordination Games

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Abstract. In the context of a "beauty contest" coordination game (in which payoffs depend on the proximity of actions to an unobserved state variable and to the average action) players choose how much costly attention to pay to various informative signals; they endogenously select information sources and how carefully to listen to them. Each signal has an underlying accuracy (how precisely it identifies the state variable) and a clarity (how easy it is for players to understand what the signal says). The unique information-acquisition equilibrium has interesting properties: only a subset of signals are assigned positive weight and attention; these are the clearest signals available, even if such signals have poor underlying accuracy; the size of the subset shrinks as the complementarity of players' actions becomes more acute; and, if actions are more complementary, the information endogenously acquired in equilibrium is more public in nature. **JEL Classifications**. C72, D83.

1. Coordination and Information Acquisition

In a "beauty contest" game the participants' payoffs depend on the proximity of their actions to an unobserved underlying state variable and to an aggregate measure of all actions.[2] When players wish to take actions close to the average action then such games have a natural interpretation: players would like to do the right thing, and do it together. Players may have different information about the identity of the state variable, and so differences of opinion may frustrate coordination.

Beauty-contest games have received close attention following the contribution of Morris and Shin (2002). Such games have been applied to complementary investments (Angeletos and Pavan, 2004), to monopolistic competition amongst heterogeneously informed firms (Hellwig, 2005), to financial markets (Allen, Morris, and Shin, 2006), to a range of other economic problems (Angeletos and Pavan, 2007), and to political leadership (Dewan and Myatt, 2008); many other papers employ variants of the beauty-contest specification. Such games are also closely related to the island-economy parable (Lucas, 1973;

[1]The authors thank colleagues, especially Torun Dewan, and seminar participants at the Universities of Warwick and Southampton for their helpful comments and suggestions.
[2]Keynes (1936, Chapter 12) described newspaper-based competitions whose entrants were invited to choose the prettiest faces from a set of photographs, but where it was optimal to nominate the most popular faces.

Phelps, 1970) so long as players are interpreted as the island sectors and their actions are market-clearing prices (Amato, Morris, and Shin, 2002; Morris and Shin, 2005). In most beauty-contest models players exogenously receive informative signals of the underlying state variable. For instance, in the model of Morris and Shin (2002) players have two information sources: one source is private (an independent signal realisation for each player) whereas the second is public (a single commonly observed signal realisation).

This paper considers endogenous information acquisition: given the availability of multiple informative but costly signals, to which signals do players listen? How carefully do they choose to listen to each? How do their equilibrium information-acquisition strategies (and, indeed, their action choices) respond to the exogenous parameters of the model? Asking these questions necessitates a step outside the established public-private taxonomy of informative signals: the amount of attention which players devote to an information source determines how "public" the corresponding signal realisations are.

More specifically, the players of a beauty-contest game are granted costly access to a collection of information sources. Each source provides an informative signal of the underlying state variable with some source-specific "sender" noise; this sender noise determines the signal's underlying quality. If a player chooses to observe a signal then it is received with some additional "receiver" noise. The receiver noise, which determines the signal's clarity, is endogenous: if a player listens with greater care (and so at greater cost) then the receiver noise is reduced. This endogenously generates arbitrary correlation across players' observations: such observations become highly correlated (the signal becomes very "public") if and only if all players pay very careful attention. More generally, the "publicity" of a signal depends on the mix of sender noise and receiver noise, with the latter endogenously determined. Having received their signal realisations players chooses their actions to maximise payoffs which reflect their dual desires to be close to the state variable and to coordinate with (or, for some parameter choices, to differ from) each other.

Allowing players to choose how carefully to observe the various information sources has important implications for the information-acquisition equilibrium. In particular, it is unique, and so comparative-static exercises are permitted; such exercises are precluded by equilibrium multiplicity. Interest lies in how actions and information-acquisition policies vary with the costliness of observing the various signals, the underlying accuracies of those signals, and the strength (and direction) of the coordination motive. Robust take-home messages emerge: only a subset of signals are assigned positive weight and attention; these are the clearest signals available, even if such signals have poor underlying accuracy; the size of the subset shrinks as the complementarity of players' actions becomes more acute; and, if actions become more complementary then the information endogenously acquired in equilibrium becomes more public in nature.

Turning back to the literature, most related research has not considered endogenous information acquisition. A notable exception is a recent article by Hellwig and Veldkamp

(2009).[3] Their appealing intuition that complementarity of action choice imposes complementarity upon information choice ("if an agent wants to do what others do, they want to know what others know") carries across to the setting here: if others pay close attention to an information source then their actions react strongly to the corresponding signal realisations; with this in mind, the coordination motive prompts a best-replying player also to react strongly to that signal; and finally, given that the signal is used heavily then it is optimal to pay close attention to it. This logic suggests that there is scope for multiple equilibria, since a player may find it optimal to listen to a signal if and only if others are expected to do so. Indeed, Hellwig and Veldkamp (2009, p. 224) argued that

> "[. . .]information choice imposes an additional requirement for equilibrium uniqueness: the information agents choose to acquire must also be private."

The idea is that the acquisition of a public signal does two things: it informs a player about the underlying state and also directly about the likely actions of others. This second effect is present if and only if others acquire the signal too, and this naturally leads to multiple equilibria. When a signal is private (so that, conditional on the underlying state, signals are independent) then the signal does not directly inform a player about the likely actions of others. This removes the key ingredient needed for multiple equilibria.

In contrast to the quoted text, this paper shows that the privacy of signals is not a requirement for uniqueness: when information sources are modelled in the way proposed here, equilibrium uniqueness is retained and therefore useful comparative-static exercises may be conducted.[4] The difference between the approach here and that taken by Hellwig and Veldkamp (2009) amounts to a difference in the interpretation of continuity in information acquisition. This seemingly small departure has striking results. The multiplicity of equilibria found by Hellwig and Veldkamp (2009) is shown to depend upon the way in which players obtain their first bit of a signal. Here players choose not only to acquire an informative signal, but also how much costly attention to pay to it. The first bit of a signal acquired (a situation in which a player pays relatively scant attention to the information source) is dominated by the aforementioned receiver noise. This ensures that the signal realisation is relatively uncorrelated with the signals received by others, and so is relatively private. Roughly speaking, this smooths out the first step of the information acquisition process and eliminates multiple equilibria, even though the informative signals actually acquired in equilibrium may be relatively public in nature.

Sections 2–3 describe the model and the unique equilibrium in which actions respond linearly to signals. Sections 4–6 show how information acquisition, actions, and the publicity of informative signals respond to the coordination motive and to other parameters. Finally, Section 7 compares the results to those of the existing literature.

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[3] Another exception is the closely related model of political leadership studied by Dewan and Myatt (2008), in which followers choose how to divide their attention between different leaders. The speeches of leaders help their followers to learn about the world and to coordinate with each other.

[4] Furthermore, the asymmetric equilibria when actions are strategic substitutes disappear.

2. A Beauty Contest with Multiple Information Sources

The model considered here is a quadratic-payoff "beauty contest" game in which players' payoffs depend upon the proximity of their actions to an unobserved underlying state variable and to the average action taken by all players. The twist is that the information sources upon which players condition their actions are both costly and endogenous.

More formally, a simultaneous-move game is played by a unit mass of players indexed by $\ell \in [0, 1]$. An individual player's move consists of the following three steps.

(1) A player chooses an information-acquisition policy $z_\ell \in \mathcal{R}^n_+$. The interpretation is that there are n information sources, and the element $z_{i\ell}$ of the vector z_ℓ is the amount of costly attention which player ℓ pays to the ith informative source.
(2) After this information-acquisition choice, the player observes a vector of n signals $x_\ell \in \mathcal{R}^n$ which inform the player about some unobserved state variable θ, where the precisions of these signals depend upon the earlier choice of z_ℓ.
(3) Finally, a player takes a real-valued signal-contingent action $a_\ell \in \mathcal{R}$.

A pure strategy is a pair $\{z_\ell, A_\ell(\cdot)\}$ where z_ℓ is the information-acquisition component and $A(x_\ell) : \mathcal{R}^n \mapsto \mathcal{R}$ specifies an action contingent on the n signal realisations.

A player's payoff depends on the proximity of the player's action a_ℓ to the underlying state variable θ, the action's proximity to the average action $\bar{a}$, and the player's information acquisition z_ℓ. Assembling these three elements, a player's payoff is

$$u_\ell = \bar{u} - \pi(a_\ell - \theta)^2 - (1 - \pi)(a_\ell - \bar{a})^2 - C(z_\ell). \tag{1}$$

The parameter $\pi \in (0, 2)$ determines a player's relative concern for targeting the state variable versus coordination with others: if $\pi = 1$ then the coordination motive is irrelevant, and note that the model allows for $\pi > 1$, in which case a player wishes to differ from others. Finally, the cost function $C(z_\ell)$ is increasing, convex, and differentiable.

It remains to specify more fully the nature of players' information sources. Players begin with no knowledge of the underlying state; without loss of generality there is an improper prior over θ. The ith signal observed by player ℓ satisfies

$$x_{i\ell} = \theta + \eta_i + \varepsilon_{i\ell}, \quad \text{where} \quad \eta_i \sim N(0, \kappa_i^2) \quad \text{and} \quad \varepsilon_{i\ell} \sim N\left(0, \frac{\xi_i^2}{z_{i\ell}}\right), \tag{2}$$

and where the various noise terms are all independently distributed.

The interpretation of (2) is that each information source has associated with it some "sender" noise η_i which reflects the quality or accuracy of an underlying signal $\bar{x}_i \equiv \theta + \eta_i$; the accuracy is indexed by the precision $1/\kappa_i^2$. A player ℓ who chooses to pay attention to the information source i does so imperfectly, owing to "receiver" noise, by observing $x_i = \bar{x}_i + \varepsilon_{i\ell}$. The receiver noise reflects the clarity with which the information is imparted, indexed by $1/\xi_i^2$, and the attention $z_{i\ell}$ that player ℓ pays to source i, so that the overall clarity of the observation is determined by the precision $z_{i\ell}/\xi_i^2$. The observation precision (or clarity) linearly increases with the choice variable $z_{i\ell}$, and so a player's information acquisition can be interpreted conveniently as a sample size. Furthermore, the choice $z_{i\ell} = 0$ is straightforwardly interpreted as the decision to ignore the ith information source completely (equivalently, the realisation $x_{i\ell}$ is pure noise in this case).

Conditional on the state θ, information sources are independent, but players' observations of each source are correlated: for two different players ℓ and ℓ', $\operatorname{cov}[x_{i\ell}, x_{i\ell'} \,|\, \theta] = \kappa_i^2$, and so observations move together unless the underlying signal $\bar{x}_i$ has perfect precision. Furthermore, the correlation of players' observations depends straightforwardly on the mix of sender noise and receiver noise. More formally, the model specification is equivalent to one in which

$$x_{i\ell} \,|\, \theta \sim N(\theta, \sigma_{i\ell}^2) \quad \text{and} \quad \operatorname{cov}[\, x_{i\ell}, x_{i\ell'} \,|\, \theta \,] = \rho_{i\ell\ell'} \times \sigma_{i\ell}\sigma_{i\ell'}, \tag{3}$$

for all $\ell' \neq \ell$ and for all i. This emerges from the specification (2) via the transformations

$$\sigma_{i\ell}^2 = \kappa_i^2 + \frac{\xi_i^2}{z_{i\ell}} \quad \text{and} \quad \rho_{i\ell\ell'} = \kappa_i^2 \left[\left(\kappa_i^2 + \frac{\xi_i^2}{z_{i\ell}} \right) \left(\kappa_i^2 + \frac{\xi_i^2}{z_{i\ell'}} \right) \right]^{-\frac{1}{2}}. \tag{4}$$

Paying more attention to an information source i (by increasing $z_{i\ell}$) not only reduces the overall variance $\sigma_{i\ell}^2$ of that signal (or, equivalently, increases the precision), but also makes it more correlated with others' observations of i (the correlation coefficient $\rho_{i\ell\ell'}$ increases).

The specification (2) and transformations (4) can be related to established models in the literature. Setting $z_{i\ell} = z_i$ for all ℓ for expositional simplicity, the correlation of players' observations of an information source is $\rho_i = \kappa_i^2/[\kappa_i^2 + (\xi_i^2/z_i)]$. The case $\rho_i = 0$, so that observations are conditionally uncorrelated, is obtained when $\kappa_i^2 = 0$, and corresponds to the "private" signal from the two-source world of Morris and Shin (2002). In contrast, the case $\rho_i = 1$, obtained in the limit as $z_i \to \infty$ or by setting $\xi_i^2 = 0$, so that players' observations coincide, corresponds to the "public" signal of Morris and Shin (2002).

For general values of κ_i^2, ξ_i^2, and z_i a signal's correlation satisfies $0 < \rho_i < 1$ so the signal is neither purely private nor purely public. As noted above, the correlation coefficient (and hence publicity of a signal) is both endogenous and also directly linked to the precision of a signal. In particular, the correlation coefficient vanishes as the attention paid to an information source shrinks to zero. What this means is that as a player begins to acquire information from a source, so that z_i moves up from zero, then the signal is initially private in nature, and only becomes more public as increasing attention is devoted to it. This feature contrasts with the specifications used by Hellwig and Veldkamp (2009). They considered a world in which players either acquire a signal or do not. This is equivalent to restricting a player's choice of $z_{i\ell}$ to take only two values. They also considered a specification in which a player's information-acquisition decision is continuous. However, that specification insists that the correlation coefficient does not change with the amount of information acquired. In the model proposed here, this is equivalent to assuming that a signal's correlation coefficient remains bounded away from zero even when hardly any attention is paid to that signal. As the discussion of Section 7 explains, it is this feature which is responsible for the presence of multiple equilibria in their model.

Before characterising equilibrium behaviour, it remains to comment on a few technical issues. Firstly, the player set is a unit mass and so each individual is negligible. This simplifies exposition, but is not crucial: appropriately modified, the results hold in a world with a finite number of players. Secondly, a player's payoff depends on the average action $\bar{a} \equiv \int_0^1 a_l \, dl$ taken across all players. Of course, this average is not always well-defined.[5] However, for the class of equilibria considered in the next section the average remains well defined both in equilibrium and following a single-player deviation. Furthermore, the specification of the game may be completed by placing payoffs on the extended real line and setting $u_\ell = -\infty$ whenever $\bar{a}$ does not exist. Thirdly, a signal's distribution is not properly specified when a player chooses $z_{i\ell} = 0$. However, this does not cause any particular problems since, as noted above, choosing $z_{i\ell} = 0$ is equivalent to ignoring an information source. Fourthly, for $\xi_i^2 > 0$ obtaining a perfectly public signal is impossible. However, this can be resolved by extending the choice of information acquisition to include $z_{i\ell} = \infty$, so long as the cost $C(z_\ell)$ is well-defined in the limit as $z_{i\ell} \to \infty$. Fifthly, players begin with no substantive prior belief over θ. However, a common prior can be accommodated easily by using one of the n signals to reflect prior beliefs, and making it costless to listen to that signal with complete attention.

3. EQUILIBRIUM

A player's strategy $\{z_\ell, A_\ell(\cdot)\}$ specifies the action $A_\ell(x_\ell)$ taken in response to each possible signal realisation x_ℓ. In principle, $A_\ell(\cdot)$ could take any form. However, there are good reasons to follow the established literature by focusing on strategies in which a player's action is a linear function of the signal realisations. To see why, suppose that all other players use a strategy $\{z, A(\cdot)\}$. Differentiating the quadratic objective function, it is straightforward to confirm that player ℓ's unique best-reply action is

$$A_\ell(x_\ell) = \pi \, \mathrm{E}[\theta \mid x_\ell] + (1 - \pi) \, \mathrm{E}[A(x_{\ell'}) \mid x_\ell], \tag{5}$$

which is a weighted average of the player's expectations of the state variable and of the average action taken by others. Given the normality assumptions, the first conditional expectation is linear in x_ℓ. If $A(\cdot)$ is linear, then the second conditional expectation is also linear in x_ℓ. Hence, if other players use a linear strategy then the unique best reply is linear. More generally, mild restrictions on the class of strategies used by players ensure that equilibrium strategies are linear.[6]

[5]For example, consider a strategy profile in which players choose actions which form a Cauchy distribution across the player set. The mean of the Cauchy does not exist, and so $\bar{a}$ is not well-defined.

[6]Morris and Shin (2002) claimed that the linear equilibrium of a beauty-contest game is unique. It has been observed (Angeletos and Pavan, 2007, fn. 5) that their logic is not completely watertight, but nevertheless it is easy to specify conditions under which only linear equilibria exist. For instance, insisting that $A(x_\ell)$ does not diverge too far from a linear strategy (so that, for some vector of weights w_ℓ, $|A(x_\ell) - \sum_{i=1}^n w_i x_{i\ell}|$

A strategy is linear if there are weights $w_\ell \in \mathcal{R}^n$ such that $A_\ell(x_\ell) = \sum_{i=1}^n w_{i\ell} x_{i\ell}$. Given linearity, a player's strategy takes the form $\{z_\ell, w_\ell\}$, and it is straightforward to confirm that $\sum_{i=1}^n w_{i\ell} = 1$, so that a player's action is a weighted average of the signals received, and $w_{i\ell}$ is the influence of the ith information source. Given that all other players employ a strategy $\{z, w\}$ then the expected payoff of a player ℓ choosing $\{z_\ell, w_\ell\}$ is

$$\mathrm{E}[u_\ell] = \bar{u} - \underbrace{\sum\nolimits_{i=1}^n w_{i\ell}^2 \left[\pi\kappa_i^2 + \frac{\xi_i^2}{z_{i\ell}}\right]}_{(\star)} - (1-\pi)\underbrace{\sum\nolimits_{i=1}^n (w_{i\ell} - w_i)^2 \kappa_i^2}_{(\dagger)} - C(z_\ell). \qquad (6)$$

Given that others play linearly (and, following the discussion in footnote 6, there is little if any loss of generality by supposing that they do) a player's best reply is to choose a pair of vectors $\{z_\ell, w_\ell\}$ to maximise (6) subject to the constraint $\sum_{i=1}^n w_{i\ell} = 1$. An inspection confirms that (6) is strictly concave, and a player's best reply is unique.

Before characterising a player's best reply and the unique symmetric linear equilibrium to the beauty-contest game, the components of (6) are discussed.

Consider each element of ($\star$). This summation is the quadratic loss experienced by a player when all player use the same weights on their signals. By placing weight on the ith information source a player is exposed to both the sender noise η_i (with variance κ_i^2) and receiver noise ε_i (with variance $\xi_i^2/z_{i\ell}$). The receiver noise, which is idiosyncratic to player ℓ, pushes the player's action away from both state variable θ and also the average action $\bar{a}$. Given that all players use the same weights, the sender noise pushes the player's action away from the state variable θ but does not push it away from the actions of others; the reason is that η_i is a common shock to all players, and so (as long as they use a common linear strategy) it has no bearing on the coordination-motive component of a player's payoff. For this reason, the variance term κ_i^2 attracts the coefficient π.

Next, consider each element of (†). This second summation is the quadratic loss experience by a player owing to the use of a different strategy from other players. Each loss here arises because of the various sender noise terms η_i. If $w_{i\ell} = w_i$ then player ℓ's action reacts to the shock η_i in the same way as other players, and so a_ℓ and $\bar{a}$ do not move apart. However, if $w_{i\ell} \neq w_i$ then owing to the different reactions the receiver noise can move a player away from others. Since this reflects the desire to coordinate (or, indeed, the desire to differ if $\pi > 1$) then (†) attracts the coefficient $1 - \pi$.

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remains bounded for all x_ℓ) is enough (Dewan and Myatt, 2008). A second approach is to consider a related game in which state, signal, and action spaces are bounded, and show that the unique equilibrium converges to the unique linear equilibrium of an unbounded game as the various bounds are removed (Calvó-Armengol, de Martí Beltran, and Prat, 2009). Finally, arguments from the classic study of team decision problems (Radner, 1962) can be used to establish a uniqueness result in finite-player beauty-contest games: for an appropriately specified finite-player version of the game considered here, and given the introduction of an appropriate proper and normal prior, the unique symmetric strategy profile which maximises the ex ante expected payoff of a randomly chosen player is the unique linear equilibrium.

Notice that (†) disappears when players use the same strategy. Furthermore, beginning from any symmetric strategy profile local changes in a player's strategy have no first-order effect on (†). This means that a strategy $\{z, w\}$ forms a symmetric equilibrium (that is, a symmetric strategy profile forming a Nash equilibrium of the game in which each player ℓ chooses $\{z_\ell, w_\ell\}$ to maximise $\mathrm{E}[u_\ell]$) if and only if $\{z, w\}$ successfully solves

$$\min \sum\nolimits_{i=1}^{n} w_i^2 \left[\pi\kappa_i^2 + \frac{\xi_i^2}{z_i}\right] + C(z_\ell) \quad \text{subject to} \quad \sum\nolimits_{i=1}^{n} w_i = 1. \tag{7}$$

The solution to this minimisation problem straightforwardly generates Proposition 1. (The proofs of this and other propositions are contained in Appendix A.)

Proposition 1. *In the unique linear symmetric equilibrium of a beauty contest game with endogenous information acquisition, each information source has influence if and only if players pay some attention to it: $w_i > 0$ if and only if $z_i > 0$. Equilibrium influence and attention satisfy*

$$w_i = \frac{\hat{\psi}_i}{\sum_{j=1}^{n} \hat{\psi}_j} \quad \textit{and} \quad z_i = \frac{\xi_i w_i}{\sqrt{C_i'(z)}}, \quad \textit{with} \quad \hat{\psi}_i = \frac{1}{\pi\kappa_i^2 + \xi_i^2/z_i}, \tag{8}$$

and where it is understood that $\hat{\psi}_i = 0$ for any information source which is ignored.

The weight attached to a particular signal is large when that signal is listened to carefully: w_i moves together with z_i. Moreover, signals have more weight attached to them whenever they are clearer or more accurate; that is, when ξ_i^2/z_i and κ_i^2 fall.

Putting aside the information-acquisition decisions for a moment, it is interesting to note that the equilibrium influence of an information source (this is determined by $\hat{\psi}_i$) depends less strongly on a signal's underlying accuracy than on its clarity whenever player's value coordination (so that $\pi < 1$). Indeed, if coordination is all that matters (so that π is close to zero) then, fixing the information-acquisition decisions, a signal's influence is proportional to its clarity. This is natural: changing a signal's underlying accuracy affects only the ability of players to hit the truth (which matters to the extent that hitting the truth matters, that is, π) whereas enhancing a signal's clarity helps players both to coordinate and also to hit the true value of θ.

The remainder of the paper examines the properties of the equilibrium described in Proposition 1 and conducts comparative-static exercises. Firstly, Section 4 analyses how the information-acquisition policy varies with the exogenous parameters, in particular the coordination preferences of the players (π). Secondly, Section 5 relates the (endogenous) publicity of information sources to the nature of comparative-static predictions. Thirdly, Section 6 examines how equilibrium actions and beliefs vary with the coordination motive. In particular, as π increases (so that coordination becomes less important to the players, and hitting the truth more important) actions become less variable around the truth and their covariance across the player set falls (as does their correlation coefficient).

4. Information Acquisition

The main focus of this paper is on the introduction of endogenous information acquisition to an otherwise-standard beauty contest, and so the determinants of z (the information-acquisition policy) are now considered. Taking (8) and substituting yields, for $z_i > 0$,

$$z_i = \frac{\xi_i(K_i - \xi_i)}{\pi\kappa_i^2} \quad \text{where} \quad K_i \equiv \frac{1}{\sqrt{\partial C(z)/\partial z_i \sum_{j=1}^n \hat{\psi}_j}}. \tag{9}$$

Treating K_i as a constant for the moment, (9) indicates that the attention paid to an information source is increasing in the quality (that is, the precision $1/\kappa_i^2$) of the underlying signal $\bar{x}_i = \theta + \eta_i$. Put more crudely, players listen more carefully to an information source whenever its provider has more to say. However, notice (again treating K_i as constant for the moment) that z_i is non-monotonic in the clarity (determined by $1/\xi_i^2$) with which the information is communicated. This is rather natural: ξ_i^2 is effectively the price of obtaining a noisy observation of $\bar{x}_i$ with precision z_i/ξ_i^2, and so z_i is a player's expenditure on that information source. This expenditure is increasing and then decreasing in the price charged. A final observation is to note that the solution given by (9) applies only so long as $\xi_i < K_i$. When ξ_i exceeds K_i then the stated solution for z_i no longer applies, and in fact $z_i = 0$. More generally, this indicates that an information source is likely to receive attention only if it is communicated with sufficient clarity.

To proceed further it proves useful to specialise on a particular form for the costs of information acquisition. Consider a world in which z_i is the time spent listening to signal i; this fits well with the interpretation of z_i as a sample size, so that the precision of the observation increases linearly with z_i. In such a world a natural specification for the cost of information is $C(z) = c(Z)$ where $Z \equiv \sum_{i=1}^n z_i$, and where $c(\cdot)$ is an increasing, convex, and differentiable cost function which reflects the opportunity cost of spending a total period of time Z gathering information. Of course, the various information sources continue to vary in their clarity, so that listening to a given signal i for some period of time longer need not reveal the same quantity of information that listening to j for the same extra time would yield. It proves convenient to label the information sources of decreasing clarity, so that $\xi_1 < \xi_2 < \ldots < \xi_n$.[7] Equivalently, higher-indexed information sources are more expensive to acquire. Note, however, that this labelling has no implications for the underlying accuracy of the information; the clearest signal may well be subject to high-variance sender noise.

Given this specific form for the cost function, the marginal cost of information acquisition is independent of i; a little more formally, $\partial C(z)/\partial z_i = c'(Z)$ for all i. Inspecting (9), this implies that K_i is equal to some constant K for all i. A direct implication is that $z_i > 0$ if and only if $\xi_i < K$: the clearest signals receive attention and consequently influence players' actions, whilst the remaining signals are ignored.

[7] Ties are excluded for convenience only. The propositions and proofs could be extended to accommodate ties (in a straightforward but cumbersome manner) but no fresh insight would be gained.

Proposition 2. *Suppose $C(z) = c(Z)$ where $Z \equiv \sum_{j=1}^{n} z_j$. There is a unique K such that*

$$z_i = \frac{\xi_i \max\{(K - \xi_i), 0\}}{\pi \kappa_i^2}. \tag{10}$$

Hence only the $m \in \{1, \dots, n\}$ clearest information sources satisfying $\xi_i < K$ receive attention and exert influence. The equilibrium satisfies these properties: (i) information sources with better accuracy ceteris paribus receive more attention; (ii) raising the marginal cost schedule reduces the attention paid to any source; and (iii) the attention paid to a source is non-monotonic in its clarity.

It is striking that not all signals receive attention: sufficient clarity is necessary (and, indeed, sufficient). Whilst clarity determines which information sources receive attention, accuracy determines—for those signals in use—how much attention each receives. Other things equal, more accurate (higher quality) signals receive more attention. Note, however, that a signal with appalling underlying accuracy (where the variance κ_i^2 of the sender noise η_i is particularly high) is nevertheless both acquired and has influence so long as the clarity with which it is communicated to players is sufficient.

This feature is usefully understood by considering the marginal benefit to increased attention. Differentiating the quadratic-loss term from (7) it is readily verified that

$$-\frac{\partial}{\partial z_i}\left[\sum_{j=1}^{n} w_j^2 \left(\pi \kappa_j^2 + \frac{\xi_j^2}{z_j}\right)\right] = \frac{w_i^2 \xi_i^2}{z_i^2} \propto \frac{1}{\xi_i^2}\left[\frac{\xi_i^2 / z_i}{\pi \kappa_i^2 + \xi_i^2 / z_i}\right]^2. \tag{11}$$

In general this marginal benefit of increased attention depends on both κ_i^2 and ξ_i^2. However, an inspection of (11) confirms that as z_i shrinks to zero this marginal benefit depends only on the clarity of the information source. Intuitively, when z_i is small the total amount of noise in an information source is dominated by the receiver noise. So, when thinking about which information source to acquire a player begins with the clearest. However, as z_i increases away from zero the marginal benefit of further attention is no longer dominated by receiver noise, and so the accuracy of the underlying signal $\bar{x}_i = \theta + \eta_i$ becomes important. This means that information sources which are clear but inaccurate are acquired, but receive only a limited attention span.

This discussion suggests that it is the properties of the first bit of a signal, as z_i rises away from zero, that determine whether an information source is used. This is also true for a second natural specification in which the cost function is additively separable, so that $C(z) = \sum_{i=1}^{n} c_i(z_i)$. It is immediate that if $c_i'(0) = 0$ then $z_i > 0$; if listening to a signal for a very short period of time adds nearly nothing to costs, it will always be worth doing so. A more interesting situation is one in which information acquisition is always costly at the margin. For this case, without loss of generality set $\xi_i^2 = \xi^2$ for all i, and label the information sources so that $c_1'(0) < c_2'(0) < \cdots < c_n'(0)$.[8] Thus the lower-indexed information sources are less costly at the margin when a player begins to bring a source into limited use. In this setting, attention is again focused on lower-indexed signals.

[8]Once again, no new insight is gained by considering the case of ties.

Proposition 3. *Suppose $C(z) = \sum_{j=1}^{n} c_j(z_j)$ and that $c_1'(0) < \cdots < c_n'(0)$. There is a unique $m \in \{1, \ldots, n\}$ such that only the first m signals receive attention and exert influence.*

Related results also hold. For instance, it is natural to say that information source i is cheaper at the margin than j if the marginal-cost schedule for i lies everywhere below that for j. If this is the case, and if signal i has better underlying accuracy than j, then of course signal i attracts more attention (and gains more influence) than signal j.

Propositions 2 and 3 establish the basic properties of equilibrium information-acquisition policies. A feature is that players may restrict attention to a subset of information sources; those which attract attention are the clearest (as in Proposition 2) or, relatedly, the cheapest to acquire (Proposition 3). An immediate question is this: how does the size of the attention-grabbing set of signals vary with the various exogenous properties of the players' environment? To answer this question it is convenient to return to the cost specification $C(z) = c(Z)$ where $Z = \sum_{j=1}^{n} z_j$, so that z_i can be interpreted as the time devoted to observing the ith information source, as well as considering (for two of the claims within the next proposition) the additive-cost case where $C(z) = \sum_{j=1}^{n} c_j(z_j)$.

Proposition 4. *Suppose that $C(z) = c(Z)$ where $Z = \sum_{j=1}^{n} z_j$. The number of information sources which attract attention (i) rises as the preference for coordination diminishes; (ii) falls as the accuracy of information sources improves; and (iii) rises as the marginal cost of information acquisition falls. If instead $C(z) = \sum_{j=1}^{n} c_j(z_j)$, then claims (i) and (ii) continue to hold.*

Note that claim (iii) concerns a simultaneous change in the cost function affecting all of the information sources, and so is particular to the specification $C(z) = c(\sum_{j=1}^{n} z_j)$. Reducing the marginal cost of information acquisition (shifting down the schedule $c'(\cdot)$) is equivalent to increasing simultaneously the clarity of all information sources by scaling down the ξ_i^2 terms. Whereas this has a predictable monotonic effect on the number of information sources which are acquired, this is not the case when the clarities are changed individually: fixing ξ_j^2 for $j \neq i$, the size m of the attention-receiving set is generally non-monotonic in ξ_i^2.[9] This is perhaps unsurprising, given that the relationship between z_i and ξ_i^2 is also non-monotonic, as an inspection of (10) confirms.

The first comparative-static prediction of Proposition 4 is, perhaps, the most interesting: as actions become less complementary (π increases) players obtain a more diverse set of information sources. On the other hand, the desire to coordinate drives players towards a smaller set of signals. As the coordination motive becomes particularly strong, so that π

[9] Consider a world in which $n = 2$ and where $m = 1$; given that $\xi_1^2 < \xi_2^2$ it is always possible to construct such a scenario by choosing π sufficiently small (as Proposition 5 below confirms). Clearly, increasing ξ_1^2 up to ξ_2^2 will raise m, as certainly both information sources are acquired whenever their clarities are equal. Also, when ξ_1^2 is lowered toward zero then m also rises. (Technically, some other conditions need to be imposed for this to be true. A sufficient condition is to impose an Inada condition on $c'(\cdot)$ by supposing that $c'(0) = 0$.) The reason is that the first signal becomes almost free to listen to without almost perfect clarity, which reduces z_1 and so lowers the marginal cost of paying attention to the second information sources. Drawing these observations together, there is no monotonic relationship between m and ξ_i^2.

shrinks toward zero, then m falls toward its minimum value. Indeed, it turns out that in this case players coordinate by paying attention only to the clearest information source.

Proposition 5. *Suppose $C(z) = c(\sum_{j=1}^{n} z_j)$. When coordination becomes sufficiently important, one signal only receives positive attention: there exists a $\hat{\pi} > 0$ such that if $\pi < \hat{\pi}$ then $m = 1$.*

Note that this result does not apply when the additive-cost specification $C(z) = \sum_{j=1}^{n} c_j(z_j)$ is used. The reason is that when π is small the marginal benefit of increased information acquisition is determined solely by a signal's clarity $1/\xi_i^2$; to see this, inspect (11). Under the additive-attention specification the marginal cost of information acquisition is equal across signals. Thus, if additional attention is devoted to an information source then, for π close to zero, this attention will be given to the clearest information source. However, under the additive-cost specification the marginal cost of acquisition generally differs across signals, and so this argument no longer works.

Propositions 4 and 5 reveal the properties of the cardinality m of the set of information sources which receive attention. They do not, however, reveal the amount of attention paid to each source as parameters change. Although more signals are acquired as the coordination motive lessens and as the underlying accuracy of signals falls, it is not the case that each signal receives more individual attention. Indeed, for many cost specifications (including those considered in this section) any change in accuracy or the coordination motive which raises the attention given to one information source must necessarily reduce the attention paid to another.[10] Before describing how the pattern of attention changes, however, it is useful to pause and consider the notion of a signal's publicity.

5. Publicity and Information Acquisition

Many contributions to the "beauty contest" literature have specified signals that are labelled as either public (perfectly correlated signal realisations) or private (uncorrelated realisations). Here, and as already suggested in Section 2, the correlation coefficient can index the general "publicity" of a signal. In equilibrium, and using an obvious notation, the correlation between two players' observations of an information source is

$$\rho_i = \frac{\kappa_i^2}{\kappa_i^2 + (\xi_i^2/z_i)}, \tag{12}$$

where z_i is the equilibrium attention paid to a signal. (For a signal which is ignored, it is convenient to set $\rho_i = 0$, which is the limit as $z_i \to 0$.) Two features distinguish our modelling framework from existing work: firstly, the publicity of an information source can and does take intermediate values; and secondly, that publicity is endogenous.

Once the (endogenous) publicities of signals (via their correlation coefficients) have been established, it is straightforward to explain how the pattern of attention paid to information sources changes with the players' desire for coordination. Intuitively, relatively

[10] This statement holds, for instance, whenever the cost function satisfies $\partial^2 C(z)/\partial z_i \partial z_j \geq 0$.

public signals act as effective focal points for players coordination. As the desire for coordination weakens (π rises) such signals become less influential and so the attention paid to them falls. In tandem, the attention paid to relatively private signals grows. This intuition is confirmed (at least for the leading cost specifications of interest) by the next proposition, which also describes the effect of changing signal accuracy.

Proposition 6. *Suppose that either $C(z) = c(\sum_{j=1}^{n} z_j)$ or $C(z) = \sum_{j=1}^{n} c_j(z_j)$. As the desire for coordination diminishes (so that π rises) attention moves away from more public signals and toward more private signals: there is a $\hat{\rho}$ such that the attention paid to a signal is decreasing in π if and only if $\rho_i > \hat{\rho}$. An increase in the underlying accuracy of a signal (a fall in κ_i^2) reduces the attention paid to it, while increasing the attention paid to all other signals.*

The final comparative-static prediction is rather obvious: attention falls away from poorer quality information sources. The effect of the coordination motive is more interesting, however: the change in the attention paid to an information source depends upon the associated signal's publicity, but this publicity is itself endogenous. In particular, as π increases attention moves away from relatively public (highly correlated) signals and so, as an inspection of (12) confirms, those signals become less correlated and so less public. In general, the attenuation in the coordination motive means that, heuristically at least, the entire set of signals becomes more "averagely public" in nature.

The results recorded in Proposition 6 take as given the equilibrium correlation coefficient (and hence publicity) of each signal. Since such coefficients are endogenous, it is interesting to consider how the exogenous properties of an information source, namely its underlying accuracy and its clarity, determine its equilibrium publicity. Given the use of the additive-attention specification for costs, this is readily determined.

Proposition 7. *Suppose $C(z) = c(Z)$ where $Z = \sum_{j=1}^{n} z_j$. In equilibrium the clearest signals are also the most public: if $\xi_i^2 < \xi_j^2$ then $\rho_i \geq \rho_j$. Hence, as the coordination motive weakens, attention moves from the clearest signals to less clear signals. Total attention $Z \equiv \sum_{j=1}^{n} z_j$ is increasing in π, and so players spend more on information acquisition as their desire to coordinate falls.*

All of the comparative-static results relating to π may be recast in terms of the accuracy of the underlying signals. From (8), it is clear that scaling up all of the κ_i^2s proportionately is equivalent to increasing π (κ_i^2 and π enter as a product and only in the expression for $\hat{\psi}_i$). Thus, it is straightforward to deal with a general reduction in the accuracy of information. For example, from Proposition 7, increasing all n of the κ_i^2s proportionately (equivalently reducing their accuracy) will (i) move attention away from the clearest signals and toward the less clear, (ii) increase the total attention paid, and so (iii) increase players' expenditure on information acquisition. This latter observation is interesting: a general decrease in signal accuracy results in higher expenditure. A reduction in the (exogenous) general level of accuracy increases the marginal benefits generated by any (endogenous) increase in clarity and hence induces players to pay more heed overall.

6. Equilibrium Actions and Beliefs

Having established some properties of players' information acquisition, consideration is now given to the beliefs which are induced and the subsequent actions which are taken.

On average each action matches the underlying state: $\mathrm{E}[a_\ell \mid \theta] = \theta$. However, actions vary, and the extent to which players succeed in hitting θ is measured by the variance $\mathrm{var}[a_\ell \mid \theta]$. Further measures of players' overall performance include the pairwise covariance $\mathrm{cov}[a_\ell, a_{\ell'} \mid \theta]$ and pairwise correlation coefficient $\mathrm{cov}[a_\ell, a_{\ell'} \mid \theta] / \mathrm{var}[a_\ell \mid \theta]$ of their actions. If more structure is imposed on the cost function then these three measures (variance, covariance, and correlation) all move together as the players' desire for coordination is changed. The specification imposed here is linear: $C(z) \propto \sum_{j=1}^{n} z_j$, which is equivalent to imposing a constant marginal cost of a player's time in a world where z_i is interpreted as the time spent listening to an information source. This functional form greatly simplifies the solution for K used in (10) and generates the following proposition.

Proposition 8. *Suppose $C(z) = \text{constant} \times \sum_{j=1}^{n} z_j$. The variance, covariance, and correlation coefficient of players' actions all rise with the players' concern for coordination.*

As the truth becomes more important, and coordination less so (so that π rises) the correlation between players' actions falls, but they take actions that vary little around θ. Moreover, this result continues to apply as π exceeds one. That is, if players are interested in doing what others do not, they will take increasingly uncorrelated actions (but based on the same information sources). This is despite the fact that, for large π, the very strong preference to hit θ drives the variability of actions around θ down.

The properties of players' posterior beliefs also change with the coordination motive. Previous results have shown that as π increases, players listen to more signals, listen for longer, and shift their attention away from the clearer information sources. However, it remains to establish what this means for posterior beliefs. It is natural to examine the conditional expectation of θ given the information acquired: that is, $\mathrm{E}[\theta \mid x_\ell]$. The following proposition begins with the variance of this expectation.

Proposition 9. *Suppose $C(z) = \text{constant} \times \sum_{j=1}^{n} z_j$. The variance of conditional expectations formed about θ (that is, $\mathrm{var}[\mathrm{E}[\theta \mid x_\ell] \mid \theta]$) decreases with π: as players become more concerned with the truth and less concerned with coordination, they obtain better (higher precision) information.*

In this sense, players' equilibrium information-acquisition policies result in more information as the coordination motive is reduced. When players care more about the truth, they obtain more accurate composite signals of the value of θ. However, and simultaneously, the information acquired in equilibrium becomes increasingly "private" in the sense that the composite information about θ obtained by each player exhibits a decreasing covariance with the information received by others.

Proposition 10. *Suppose $C(z) = constant \times \sum_{j=1}^{n} z_j$. If coordination is sufficiently unimportant ($\pi > \frac{1}{2}$) or there are not too many information sources ($n \leq 3$), the covariance of conditional expectations formed about θ decreases with π: as players become more concerned with the truth and less concerned with coordination, they obtain less "public" and more "private" information.*

This reinforces how the public-private distinction present in much of the previous literature might miss an important arc of the story: this distinction is an output rather than input. If players endogenously choose which sort of information to acquire, the properties of the aggregate information obtained in equilibrium will depend upon the strength of their coordination motive. As the motive is reduced the information acquired endogenously becomes more private—the covariance between any two players' composite signals about the underlying parameter θ is reduced. Once again, publicity is usefully thought of as an endogenous property.

7. Related Literature

Researchers including Morris and Shin (2002, 2005), Hellwig (2005), and Angeletos and Pavan (2004, 2007) have studied models in which the players of beauty-contest games have exogenous access to information sources; for most papers (although not all) such informative signals are either "public" or "private" in nature. This paper contributes in two ways: firstly, it allows for endogenous information acquisition; and secondly, it allows that acquisition to change the nature (in particular, the publicity as well as the precision) of the signals. Other recent papers have also considered endogenous information acquisition, and so this section discusses work by Dewan and Myatt (2008), Hellwig and Veldkamp (2009), and Calvó-Armengol, de Martí Beltran, and Prat (2009).

The model of Dewan and Myatt (2008) is closely related to this one; many of their results are special cases of those presented here. They used a beauty-contest game as a metaphor for a political party. Party members must advocate a policy, and in so doing want to do the right thing for the party (a policy close to θ) while preserving party unity (a policy close to the "party line"). Before making their decisions they listen to leaders. These leaders personify information sources. Party members can divide a fixed unit of time between listening to different leaders. Their model is equivalent to a special case of the one presented here: their cost function takes the form $C(z) = c(Z)$ for $Z = \sum_{j=1}^{n} z_j$ where costs are zero if $Z \leq 1$ and infinite (or, at least, sufficiently large) otherwise.[11]

Perhaps most interestingly, this paper offers a complementary perspective to the messages of Hellwig and Veldkamp (2009). Their first main result shows that the incentive for players to acquire information is enhanced when others acquire information and when actions are complementary; as their title suggests, players want to know what others know whenever they want to do what others do. They then considered information-acquisition

[11] Dewan and Myatt (2008) also allowed the properties of information sources to be endogenous by considering the rhetorical strategies of leaders: such leaders vary their clarities (ξ_i^2) in order to attract attention.

equilibria, and the main message from that work is that there can be many equilibria. For instance, when players are faced with a choice between acquiring a signal or not, there may be an equilibrium in which everyone acquires the signal and another equilibrium in which everyone ignores it; this seems natural given the complementarity inherent in information acquisition. This result survives even when information acquisition is "near continuous" so that the precision of the signal in question and its cost are both small. This contrasts noticeably with the findings of this paper, in which the equilibrium is unique. So what explains the difference in the messages of their paper and this one?

Technically, Hellwig and Veldkamp (2009) used a specification in which the cost of information acquisition is non-convex.[12] More importantly, however, the publicity of a signal (its correlation coefficient) is exogenous in their world. Even if only a small amount is spent on information acquisition, so that the extra precision obtained and the extra cost incurred are both small, the correlation coefficient is bounded away from zero. For instance, a player can acquire a very small bit of a public signal. Heuristically, at least, such a public signal is more valuable if others are acquiring it too; this is the source of the multiplicity. As Hellwig and Veldkamp (2009) correctly observed, this does not happen with private signals: when signal realisations are uncorrelated, there is a unique equilibrium.

This paper offers a more nuanced view. Here, as a player pays more attention to an information source (so that z_i grows) then the correlation coefficient of the signal realisation rises too; an inspection of (4) confirms this. Hence the publicity of a signal, as well as its precision, is under the control of the acquiring player. Thus, implicitly at least, this model endogenises the nature of acquisition as well as the decision to acquire; players choose how carefully to listen to their sources. Crucially, the first bit of a signal acquired is private in nature: the correlation coefficient falls to zero as z_i vanishes. This smoothes things out sufficiently to ensure that there is a unique equilibrium. Thus, when Hellwig and Veldkamp (2009, p. 224) stated that a requirement for uniqueness is that "the information agents choose to acquire must also be private" they were correct only when the decision is to acquire or not; if players choose how carefully to listen then the crucial feature is that the signal is almost perfectly private when a player pays almost no attention to it. The move from a model with multiple equilibria to one with a unique equilibrium is of interest because it allows for rich comparative-static exercises. Whereas Hellwig and Veldkamp (2009) offered the knowing what others know and the multiple-equilibria messages, here the uniqueness of the equilibrium allows specific predictions about what kind of information is acquired and how acquisition decisions change with both the nature of the information and the nature of the coordination problem faced by the players.

[12] In their world a player either acquires a signal or not. For an acquired signal the variance of the receiver noise is fixed. This is equivalent to specifying, for some $\bar{z}_i$, a cost function of the form:

$$c_i(z_i) = \begin{cases} 0 & z_i = 0 \\ \bar{c}_i & 0 < z_i \leq \bar{z}_i \\ \infty & \bar{z}_i < z_i. \end{cases}$$

It is the non-convexity of this cost function which is (technically) the source of multiple equilibria.

A final related and interesting strand of literature is the work of Calvó-Armengol and de Martí Beltran (2007, 2009) and particularly Calvó-Armengol, de Martí Beltran, and Prat (2009). In these papers a set of players arranged on a network share information they hold concerning the state of the world with others they are linked to on the network, before playing a beauty contest of the sort studied above. The papers study the impact that the network structure has upon the spread of actions in the game where (in the first two papers) that network structure is exogenous and (in the third paper) the players themselves decide whether to form pairwise links with other players at some cost, thereby endogenising the network structure and hence the information acquired. Given that extant information is passed between players, the focus of this work is elsewhere, however it is related to the current paper to the extent that information acquisition is endogenous and co-determined with the actions of the underlying beauty contest.

Appendix A. Omitted Proofs

In the main text it is claimed that any linear equilibrium strategy satisfies $\sum_{i=1}^{n} w_i = 1$. To see why, consider a linear equilibrium strategy profile $A(x_\ell) = w'x_\ell$, where w' is the transpose of $w \in \mathcal{R}^n$. Given the linearity, $\mathrm{E}[A(x_{\ell'}) \,|\, x_\ell] = w' \,\mathrm{E}[x_{\ell'} \,|\, x_\ell]$. Given normality, the latter conditional expectation satisfies $\mathrm{E}[x_{\ell'} \,|\, x_\ell] = Bx_\ell$ where B is a $n \times n$ inference matrix with the property that the rows of B sum to one. Similarly, $\mathrm{E}[\theta \,|\, x_\ell] = a'x_\ell$ where the elements of $a \in \mathcal{R}^n$ also sum to one. Using (5), $w'x_\ell = \pi \,\mathrm{E}[\theta \,|\, x_\ell] + (1-\pi)\,\mathrm{E}[A(x_{\ell'}) \,|\, x_\ell] = [\pi a + (1-\pi)B'w]'x_\ell$, and hence $w = \pi a + (1-\pi)B'w$. Given that the elements of a sum to one and each column of B' sums to one, this equality can only hold if the elements of w sum to one.

Calculation of (6). $a_\ell - \theta = \sum_{i=1}^{n} w_{i\ell}(\eta_i + \varepsilon_{i\ell})$, and so

$$\mathrm{E}[(a_\ell - \theta)^2] = \sum\nolimits_{i=1}^{n} w_{i\ell}^2 \left(\kappa_i^2 + \frac{\xi_i^2}{z_{i\ell}}\right). \tag{13}$$

The average action is $\bar{a} = \theta + \sum_{i=1}^{n} \eta_i$, since the individual-specific errors disappear via the law of large numbers and so $a_\ell - \bar{a} = \sum_{i=1}^{n} w_{i\ell}\varepsilon_{i\ell} + \sum_{i=1}^{n}(w_{i\ell} - w_i)\eta_i$. Hence:

$$\mathrm{E}[(a_\ell - \bar{a})^2] = \sum\nolimits_{i=1}^{n} \frac{w_{i\ell}^2 \xi_i^2}{z_{i\ell}} + \sum\nolimits_{i=1}^{n} (w_{i\ell} - w_i)^2 \kappa_i^2. \tag{14}$$

Substituting these two expressions yields the expression for $\mathrm{E}[u_\ell]$ given in (6). □

Proof of Proposition 1. The expression for z_i can be obtained from the first-order condition with respect to z_i. To obtain the solutions for the influence weights w, fix z and note that the optimisation problem is to minimise $L \equiv \sum_{i=1}^{n}(w_i^2/\hat{\psi}_i)$ subject to $\sum_{i=1}^{n} w_i = 1$. A solution must satisfy $\partial L/\partial w_i = \partial L/\partial w_j$ for all $i \neq j$, which holds if and only if $w_i \propto \hat{\psi}_i$. □

Calculation of (9). Once the weights w (from Proposition 1) have been obtained and substituted into the objective function, the solution for z emerges by minimising $L(z) + C(z)$ where

$$L(z) \equiv \frac{1}{\sum_{i=1}^{n} \hat{\psi}_i} \quad \text{and where} \quad \hat{\psi}_i \equiv \frac{1}{\pi\kappa_i^2 + \xi_i^2/z_i}. \tag{15}$$

For $z_i > 0$ the first-order condition with respect to z_i takes the form

$$-\frac{\partial L(z)}{\partial z_i} = \frac{\partial C(z)}{\partial z_i} \quad \Leftrightarrow \quad \frac{\xi_i^2}{(\pi\kappa_i^2 z_i + \xi_i^2)^2} = \left(\sum\nolimits_{j=1}^{n} \hat{\psi}_j\right)^2 \frac{\partial C(z)}{\partial z_i} \equiv \frac{1}{K_i^2}, \tag{16}$$

which can be re-arranged to yield (9). (Note that the first-order condition can hold only if $\xi_i < K_i$. Furthermore, a solution to the minimisation problem also requires $\xi_i \geq K_i$ whenever $z_i = 0$.) □

Proof of Proposition 2. $\partial C(z)/\partial z_i = c'(Z)$ for all i and so $K_i = K$ for all i. The calculation of (9) noted that $\xi_i < K_i$ when $z_i > 0$ and $\xi_i \geq K_i$ when $z_i = 0$. Given that that $K_i = K$ for all i, this implies that the information sources attracting attention are those with the lowest ξ_i. This yields the first claim. Substituting the expression for z_i from (9) into $\hat{\psi}_i$ yields

$$\hat{\psi}_i = \frac{1}{\pi\kappa_i^2}\left(1 - \frac{\xi_i}{K}\right) \quad \Rightarrow \quad \sum\nolimits_{j=1}^{n} \hat{\psi}_j = \frac{1}{\pi K}\sum\nolimits_{j=1}^{m} \frac{(K-\xi_j)}{\kappa_j^2} = \frac{1}{\pi K}\sum\nolimits_{j=1}^{n} \frac{\max\{(K-\xi_j), 0\}}{\kappa_j^2}. \tag{17}$$

The second part of (9) yields $1/K = \sqrt{c'(Z)}\sum_{j=1}^{n} \hat{\psi}_j$. Combining this with (17):

$$c'\left(\sum\nolimits_{j=1}^{n} \frac{\xi_j \max\{(K-\xi_j), 0\}}{\pi\kappa_j^2}\right)\left(\sum\nolimits_{j=1}^{n} \frac{\max\{(K-\xi_j), 0\}}{\pi\kappa_j^2}\right)^2 = 1. \tag{18}$$

The left-hand side of (18) is increasing in K, and so (18) yields a unique solution for K. This can be used to obtain the solution for the individual attention levels paid to each information source.

Claims (i) and (iii) in the proposition follow by inspection. Claim (ii) is obtained by observing that anything which increases the left-hand side of (18) must reduce the solution K and hence reduce m. By inspection, raising the marginal cost schedule $c'(\cdot)$ does this. □

Proof of Proposition 3. Contrary to the proposition, suppose that players ignore the ith information source (so that $z_i = 0$) while listening to source $i+1$ (so that $z_{i+1} > 0$). Now

$$\xi < K_{i+1} = \frac{1}{\sqrt{c'_{i+1}(z_{i+1})}\sum_{j=1}^{n} \hat{\psi}_j} \leq \frac{1}{\sqrt{c'_{i+1}(0)}\sum_{j=1}^{n} \hat{\psi}_j} < \frac{1}{\sqrt{c'_i(0)}\sum_{j=1}^{n} \hat{\psi}_j} = K_i. \tag{19}$$

The first inequality holds because $z_{i+1} > 0$; the second is from the convexity of $c_{i+1}(\cdot)$; and the third inequality holds by assumption. This implies $\xi < K_i$, which contradicts $z_i = 0$. □

Proof of Proposition 4. If $C(z) = c(Z)$ where $Z \equiv \sum_{j=1}^{n} z_j$ then Proposition 2 applies, where K is the unique solution to (18). The left-hand side of (18) is decreasing in π and κ_i^2 for each i, and also falls as the schedule $c'(\cdot)$ falls. Hence the solution K (and so the number m of information sources which attract attention) increases with π and κ_i for each i but falls as $c'(\cdot)$ rises.

Turning to the specification $C(z) = \sum_{j=1}^{n} c_j(z_j)$ combine the equalities from (9) to obtain

$$z_i = \frac{\xi_i}{\pi\kappa_i^2}\max\left\{\left(\frac{1}{\Psi\sqrt{c'_i(z_i)}} - \xi_i\right), 0\right\} \quad \text{where} \quad \Psi \equiv \sum\nolimits_{j=1}^{n} \hat{\psi}_j, \tag{20}$$

where this formulation also holds for $z_i = 0$. Treating Ψ as a constant, the right-hand side of the first equation in (20) is decreasing in z_i and so (20) yields a unique solution $z_i = f_i(\pi, \kappa_i^2, \xi_i, \Psi)$ for some function $f_i(\cdot)$. That solution is decreasing in π, κ_i^2, and Ψ. Given this solution for each i, the second equation in (20) can be written

$$\Psi = \sum_{j=1}^{n} \frac{1}{\pi\kappa_j^2 + [\xi_j^2/f_j(\pi, \kappa_j^2, \xi_j, \Psi)]}. \tag{21}$$

Given the observations made so far, the right-hand side of this equation is decreasing in Ψ, and so (21) yields a unique solution for Ψ. The right-hand side is also decreasing in π and in κ_j^2 for each j, and so the solution Ψ is decreasing in these parameters. This property of Ψ is enough to establish claims (i) and (ii). To see why, inspect (20) and notice that an information source i is ignored if and only if $\xi_i\Psi\sqrt{c_i'(0)} > 1$. If π or κ_j^2 is reduced, then the consequent increase in Ψ strengthens this inequality and so information source i continues to be ignored. Claims (i) and (ii) follow. □

Proof of Proposition 5. The proof of Proposition 2 established that K is increasing in π. Hence, as the coordination motive is allowed to dominate, so that π approaches zero from above, K converges to a lower bound $\bar{K}$. If $\bar{K} > \xi_1$ then the left-hand side of (18) diverges, and so the equality cannot hold. Hence it must be the case that $\bar{K} = \xi_1$, which means that K must fall below ξ_2 for π sufficiently small. This in turns means that $m = 1$ for π close enough to zero. □

Proof of Proposition 6. Consider the cost specification $C(z) = c(\sum_{j=1}^{n} z_j)$ and an information source satisfying $z_i > 0$. Differentiate the solution for z_i stated in Proposition 2 to obtain

$$\frac{dz_i}{d\pi} = -\frac{\xi_i(K-\xi_i)}{\pi^2\kappa_i^2} + \frac{\xi_i}{\pi\kappa_i^2}\frac{dK}{d\pi} > 0 \quad \Leftrightarrow \quad \xi_i > K - \pi\frac{dK}{d\pi}, \tag{22}$$

and so attention grows with π if and only if the clarity of an information source is sufficiently poor. However, in equilibrium the correlation coefficient ρ_i of a signal is monotonic in its clarity:

$$\rho_i = \frac{\kappa_i^2}{\kappa_i^2 + \xi_i^2/z_i} = \frac{K-\xi_i}{K-(1-\pi)\xi_i}. \tag{23}$$

Turning to the specification $C(z) = \sum_{j=1}^{n} c_j(z_j)$, use (20) for $z_i > 0$ to obtain

$$z_i = \frac{\xi_i}{\pi\kappa_i^2}\left(\frac{1}{\Psi\sqrt{c_i'(z_i)}} - \xi_i\right). \tag{24}$$

Now, z_i is increasing in π if and only the right-hand side is increasing in π when z_i is fixed. Differentiating the right-hand side yields

$$\begin{aligned}\frac{\partial}{\partial\pi}\left[\frac{\xi_i}{\pi\kappa_i^2}\left(\frac{1}{\Psi\sqrt{c_i'(z_i)}} - \xi_i\right)\right] &= -\frac{\xi_i}{\pi^2\kappa_i^2}\left(\frac{1}{\Psi\sqrt{c_i'(z_i)}} - \xi_i\right) - \frac{\xi_i}{\pi\kappa_i^2}\left(\frac{1}{\Psi^2\sqrt{c_i'(z_i)}}\right)\frac{\partial\Psi}{\partial\pi} \\ &= -\frac{z_i}{\pi} - \left(z_i + \frac{\xi_i^2}{\pi\kappa_i^2}\right)\frac{\partial\log\Psi}{\partial\pi} > 0 \quad \Leftrightarrow \quad 1 + \left(\pi + \frac{\xi_i^2/z_i}{\kappa_i^2}\right)\frac{\partial\log\Psi}{\partial\pi} < 0. \end{aligned} \tag{25}$$

The term specific to i is monotonic in the correlation coefficient $\rho_i = \kappa_i^2/(\kappa_i^2 + \xi_i^2/z_i)$. Specifically:

$$\frac{dz_i}{d\pi} > 0 \quad \Leftrightarrow \quad \left(\pi + \frac{1-\rho_i}{\rho_i}\right)\frac{\partial\log\Psi}{\partial\pi} < -1 \quad \Leftrightarrow \quad \left(\pi + \frac{1-\rho_i}{\rho_i}\right)\left|\frac{\partial\log\Psi}{\partial\pi}\right| > 1, \tag{26}$$

where the last step uses the fact that Ψ is decreasing in π. This final equality holds if and only if ρ_i is sufficiently small; that is, if and only if the information source is relatively private. □

Proof of Proposition 7. The first claims follow from arguments given in the proof of Proposition 6. To establish that $Z = \sum_{j=1}^{n}$ is increasing in π, suppose (for the purpose of contradiction) that it is not. Summing the expression in (22) for $dz_i/d\pi$ across the m active information sources and re-arranging, total attention Z is decreasing in π if and only if

$$\sum_{j=1}^{m} \frac{\xi_j(K-\xi_j)}{\kappa_j^2} > \pi \frac{dK}{d\pi} \sum_{j=1}^{m} \frac{\xi_j}{\kappa_j^2}. \tag{27}$$

Inspecting (18), note that Z is the argument of the $c'(\cdot)$ term. Hence if Z is decreasing in π then the squared term must be increasing in π. This is so if and only if

$$\sum_{j=1}^{m} \frac{K-\xi_j}{\kappa_j^2} < \pi \frac{dK}{d\pi} \sum_{j=1}^{m} \frac{1}{\kappa_j^2}. \tag{28}$$

Combining the two inequalities of (27) and (28) gives the single inequality

$$\begin{aligned}
&\sum_{i=1}^{m}\sum_{j=1}^{m} \frac{\xi_j(K-\xi_j)}{\kappa_i^2\kappa_j^2} > \sum_{i=1}^{m}\sum_{j=1}^{m} \frac{\xi_j(K-\xi_i)}{\kappa_i^2\kappa_j^2} \\
&\quad\Leftrightarrow \quad \sum_{i\neq j} \frac{\xi_j(K-\xi_j)+\xi_i(K-\xi_i)}{\kappa_i^2\kappa_j^2} > \sum_{i\neq j} \frac{\xi_j(K-\xi_i)+\xi_i(K-\xi_j)}{\kappa_i^2\kappa_j^2} \\
&\quad\quad\Leftrightarrow \quad 0 > \sum_{i\neq j} \frac{\xi_i^2+\xi_j^2-2\xi_i\xi_j}{\kappa_i^2\kappa_j^2} = \sum_{i\neq j} \frac{(\xi_i-\xi_j)^2}{\kappa_i^2\kappa_j^2}.
\end{aligned} \tag{29}$$

The final expression is positive, which generates the desired contradiction. □

Proof of Proposition 8. Setting $C(z) = \gamma \sum_{i=1}^{n} z_i$, $K_i = K$ where $1/K = \sqrt{\gamma}\sum_{i=1}^{n} \hat{\psi}_i$. Algebra yields

$$w_i = \frac{\sqrt{\gamma}}{\pi}\frac{\max\{(K-\xi_i),0\}}{\kappa_i^2} \quad \text{and} \quad \sigma_i^2 \equiv \kappa_i^2 + \frac{\xi_i^2}{z_i} = \kappa_i^2\left[\frac{K-(1-\pi)\xi_i}{K-\xi_i}\right], \tag{30}$$

where the solution for σ_i^2 applies and is needed only for $i \leq m$. Hence

$$\text{var}[a_\ell \,|\, \theta] = \sum_{i=1}^{m} w_i^2\sigma_i^2 = \frac{\gamma}{\pi^2}\sum_{i=1}^{m} \frac{(K-\xi_i)(K-(1-\pi)\xi_i)}{\kappa_i^2}. \tag{31}$$

Given the cost assumptions, the equation (18) determining K becomes

$$\gamma\left(\sum_{j=1}^{m} \frac{K-\xi_j}{\pi\kappa_j^2}\right)^2 = 1 \quad \Rightarrow \quad K = \bar{\xi} + \frac{\pi}{\sqrt{\gamma}\sum_{j=1}^{m}(1/\kappa_j^2)} \quad \text{where} \quad \bar{\xi} \equiv \frac{\sum_{j=1}^{m}(\xi_j/\kappa_j^2)}{\sum_{j=1}^{m}(1/\kappa_j^2)}. \tag{32}$$

Substituting K back into $\text{var}[a_\ell \,|\, \theta]$ yields, after some algebraic simplification,

$$\begin{aligned}
\text{var}[a_\ell \,|\, \theta] &= \gamma\sum_{i=1}^{m} \frac{1}{\kappa_i^2}\left(\frac{\bar{\xi}-\xi_i}{\pi} + \frac{1}{\sqrt{\gamma}\sum_{j=1}^{m}(1/\kappa_j^2)}\right)\left(\frac{\bar{\xi}-\xi_i}{\pi} + \frac{1}{\sqrt{\gamma}\sum_{j=1}^{m}(1/\kappa_j^2)} + \xi_i\right) \\
&= \bar{\xi}\sqrt{\gamma} + \frac{1}{\sum_{j=1}^{m}(1/\kappa_j^2)} + \gamma\left(\frac{1-\pi}{\pi^2}\right)\sum_{i=1}^{m}\frac{(\xi_i-\bar{\xi})^2}{\kappa_i^2}.
\end{aligned} \tag{33}$$

This is decreasing in π if $\pi < 2$, which is a maintained parameter restriction of the model. Turning to the pairwise covariance between players' actions,

$$\begin{aligned}
\text{cov}[a_\ell, a_{\ell'} \,|\, \theta] &= \sum_{i=1}^{n} w_i^2\kappa_i^2 = \frac{\gamma}{\pi^2}\sum_{i=1}^{m}\frac{(K-\xi_i)^2}{\kappa_i^2} = \frac{\gamma}{\pi^2}\sum_{i=1}^{m}\frac{1}{\kappa_i^2}\left(\frac{\pi}{\sqrt{\gamma}\sum_{j=1}^{m}(1/\kappa_j^2)} - (\xi_i-\bar{\xi})\right)^2 \\
&= \frac{1}{\sum_{j=1}^{m}(1/\kappa_j^2)} + \frac{\gamma}{\pi^2}\sum_{i=1}^{m}\frac{(\xi_i-\bar{\xi})^2}{\kappa_i^2},
\end{aligned} \tag{34}$$

where K has been substituted as before. By inspection, this covariance is decreasing in π.

The correlation coefficient of action across players (conditional on θ) is

$$\hat{\rho}_{\ell\ell'} \equiv \frac{\operatorname{cov}[a_\ell, a_{\ell'} \mid \theta]}{\operatorname{var}[a_\ell \mid \theta]} = \frac{\frac{1}{\sum_{j=1}^m (1/\kappa_j^2)} + \frac{\gamma}{\pi^2}\sum_{i=1}^n \frac{(\xi_i-\bar{\xi})^2}{\kappa_i^2}}{\bar{\xi}\sqrt{\gamma} + \frac{1}{\sum_{j=1}^m (1/\kappa_j^2)} + \gamma\left(\frac{1-\pi}{\pi^2}\right)\sum_{i=1}^m \frac{(\xi_i-\bar{\xi})^2}{\kappa_i^2}} = \frac{\pi^2 + B}{(A+1)\pi^2 + (1-\pi)B}$$

$$\text{where} \quad A \equiv \bar{\xi}\sqrt{\gamma}\sum_{j=1}^m (1/\kappa_j^2) \quad \text{and} \quad B \equiv \gamma\sum_{i=1}^m \frac{(\xi_i-\bar{\xi})^2}{\kappa_i^2}\sum_{j=1}^m \frac{1}{\kappa_j^2}. \tag{35}$$

Differentiating with respect to π:

$$\frac{d\hat{\rho}_{\ell\ell'}}{d\pi} = \frac{B^2 - \pi^2 B - 2\pi AB}{((A+1)\pi^2 + (1-\pi)B)^2} < 0 \quad \Leftrightarrow \quad B < \pi^2 + 2\pi A. \tag{36}$$

For π small enough, $m = 1$ and so $B = 0$ and so this inequality holds. Fixing A and B, the inequality strengthens as π increases. The only remaining case is when m increases following a rise in π, so that A and B both change. However, straightforward but long and tedious algebraic manipulations confirm that such an increase in m serves to strengthen the inequality. □

Proof of Proposition 9. Write $\psi_i \equiv 1/\sigma_i^2$ where $\sigma_i^2 = \kappa_i^2 + (\xi_i^2/z_i)$ for the precision of the ith signal. The precision of a player's posterior beliefs about θ is $\sum_{j=1}^m \psi_j$ and $\operatorname{var}[\mathrm{E}[\theta \mid x_\ell] \mid \theta] = 1/\sum_{j=1}^m \psi_j$. The proposition claim, therefore, is that $\sum_{j=1}^m \psi_j$ is increasing in π. Taking σ_i^2 from (30), differentiating ψ_i with respect to π, substituting in the derivative of K with respect to π obtained from differentiating the expression for K stated in (32), and re-arranging yields

$$\frac{d\psi_i}{d\pi} = \frac{\xi_i}{\kappa_i^2}\frac{\xi_i - \bar{\xi}}{(K-(1-\pi)\xi_i)^2}. \tag{37}$$

Hence $\sum_{j=1}^m \psi_j$ is increasing in π if and only if

$$\sum_{j=1}^m \frac{\xi_j}{\kappa_j^2}\frac{\xi_j - \bar{\xi}}{(K-(1-\pi)\xi_j)^2} > 0 \;\Leftrightarrow\; \sum_{j=1}^m \frac{\xi_j^2}{\kappa_j^2}\frac{1}{(K-(1-\pi)\xi_j)^2} > \bar{\xi}\times\sum_{j=1}^m \frac{\xi_j}{\kappa_j^2}\frac{1}{(K-(1-\pi)\xi_j)^2}$$

$$\Leftrightarrow \quad \sum_{k=1}^m \frac{1}{\kappa_k^2}\sum_{j=1}^m \frac{\xi_j^2}{\kappa_j^2}\frac{1}{(K-(1-\pi)\xi_j)^2} > \sum_{k=1}^m \frac{\xi_k}{\kappa_k^2}\sum_{j=1}^m \frac{\xi_j}{\kappa_j^2}\frac{1}{(K-(1-\pi)\xi_j)^2}. \tag{38}$$

This inequality is characterised by two products, the jkth elements of which cancel from both sides whenever $j = k$. Consider $j \neq k$. Collecting together the terms on either side in a typical such jkth element, a sufficient condition for the above inequality is that, for all j and k,

$$\frac{1}{\kappa_j^2\kappa_k^2}\left[\frac{\xi_j^2}{(K-(1-\pi)\xi_j)^2} + \frac{\xi_k^2}{(K-(1-\pi)\xi_k)^2}\right] > \frac{1}{\kappa_j^2\kappa_k^2}\left[\frac{\xi_j\xi_k}{(K-(1-\pi)\xi_j)^2} + \frac{\xi_j\xi_k}{(K-(1-\pi)\xi_k)^2}\right]$$

$$\Leftrightarrow \quad \frac{\xi_j(\xi_j-\xi_k)}{(K-(1-\pi)\xi_j)^2} > \frac{\xi_k(\xi_j-\xi_k)}{(K-(1-\pi)\xi_k)^2}. \tag{39}$$

Suppose first that $\xi_j > \xi_k$, then dividing by the (positive) common element simplifies this inequality to $\xi_j(K-(1-\pi)\xi_k)^2 > \xi_k(K-(1-\pi)\xi_j)^2$. Multiplying out, cancelling the common component and collecting terms again simplifies further to $(\xi_j - \xi_k)K^2 > (\xi_j - \xi_k)(1-\pi)^2\xi_j\xi_k$. Given that $\xi_j > \xi_k$ has been assumed, the first term on each side can be cancelled and the result is true if $K > \xi_j$ (for $\pi < 2$, which is assumed throughout). But, since $z_j > 0$ for such j, K is certainly larger than ξ_j. Finally, when $\xi_j < \xi_k$, the penultimate two inequalities both reverse (returning exactly the same final inequality) and the result holds once more, since $K > \xi_k$. □

Proof of Proposition 10. Set $\gamma = 1$ without loss of generality. The covariance of interest is

$$\operatorname{cov}[\mathrm{E}[\theta \mid x_\ell], \mathrm{E}[\theta \mid x_{\ell'}] \mid \theta] = \frac{\sum_{j=1}^m \psi_j^2 \, \mathrm{E}[(x_{j\ell} - \theta)(x_{j\ell'} - \theta) \mid \theta]}{(\sum_{j=1}^m \psi_j)^2} = \frac{\sum_{j=1}^m \psi_j^2 \kappa_j^2}{(\sum_{j=1}^m \psi_j)^2} = \frac{\sum_{j=1}^m \psi_j \rho_j}{(\sum_{j=1}^m \psi_j)^2}, \quad (40)$$

where the second equality follows from independence across information sources, the third by definition, and where $\rho_i \equiv \psi_i \kappa_i^2$. Simplifying notation and differentiating with respect to π gives

$$\begin{aligned}\frac{d\,\mathrm{cov}}{d\pi} &= \frac{1}{(\sum_{j=1}^m \psi_j)^2} \sum_{j=1}^m \left(\frac{d\psi_j}{d\pi}\rho_j + \psi_j \frac{d\rho_j}{d\pi} \right) - \frac{2}{(\sum_{j=1}^m \psi_j)^3} \sum_{j=1}^m \frac{d\psi_j}{d\pi} \sum_{j=1}^m \psi_j \rho_j \\ &= \frac{2}{(\sum_{j=1}^m \psi_j)^2} \sum_{j=1}^m \frac{d\psi_j}{d\pi}\left(\rho_j - \rho\right), \quad (41)\end{aligned}$$

where $\rho \equiv \sum_{j=1}^m \psi_j \rho_j / \sum_{j=1}^m \psi_j$. Therefore, the covariance decreases with π if and only if the final term above is negative. From (37), $d\psi_i/d\pi > 0$ if and only if $\xi_i > \bar{\xi}$. Again from (37),

$$\rho_i = \psi_i \kappa_i^2 = \frac{K - \xi_i}{K - (1-\pi)\xi_i} \quad \text{and so} \quad \rho_i > \rho_j \quad \Leftrightarrow \quad \xi_i < \xi_j \quad (42)$$

is confirmed by straightforward algebra. Now, the differential of the covariance can be written

$$\frac{d\,\mathrm{cov}}{d\pi} = \sum_{\xi_j < \bar{\xi}} \underbrace{\frac{d\psi_j}{d\pi}}_{-\mathrm{ve}} (\rho_j - \rho) + \sum_{\xi_j > \bar{\xi}} \underbrace{\frac{d\psi_j}{d\pi}}_{+\mathrm{ve}} (\rho_j - \rho) < \sum_{\xi_j < \bar{\xi}} \frac{d\psi_j}{d\pi} (\bar{\rho} - \rho) + \sum_{\xi_j > \bar{\xi}} \frac{d\psi_j}{d\pi} (\bar{\rho} - \rho), \quad (43)$$

where $\bar{\rho} = (K - \bar{\xi})/(K - (1-\pi)\bar{\xi})$. Thus, collecting together the terms in the summation again,

$$\frac{d\,\mathrm{cov}}{d\pi} < 0 \quad \text{if} \quad (\bar{\rho} - \rho) \sum_{j=1}^m \frac{d\psi_j}{d\pi} < 0 \quad \Leftrightarrow \quad \bar{\rho} < \rho, \quad (44)$$

where the latter statement follows from Proposition 9. Recall $\sqrt{c'} = 1$, and so, using (32) for K,

$$\bar{\rho} = \frac{K - \bar{\xi}}{K - (1-\pi)\bar{\xi}} = \frac{1}{1 + \sum_{j=1}^m \xi_j / \kappa_j^2}, \quad \text{and} \quad \rho = \frac{\sum_{j=1}^m \psi_j \rho_j}{\sum_{j=1}^m \psi_j} \quad (45)$$

by definition. So $\rho > \bar{\rho}$ if and only if $\sum_{j=1}^m \psi_j \rho_j (1 + \sum_k \xi_k/\kappa_k^2) > \sum_{j=1}^m \psi_j$. Rearranging, this occurs if and only if $\sum_{j=1}^m \psi_j (\rho_j (1 + \sum_k \xi_k/\kappa_k^2) - 1) > 0$. Using the definitions for ρ_i and for K from (32),

$$\begin{aligned}\rho > \bar{\rho} \quad &\Leftrightarrow \quad \sum_{j=1}^m \psi_j \frac{\bar{\xi} - \xi_j}{K - (1-\pi)\xi_j} > 0 \quad \Leftrightarrow \quad \sum_{j=1}^m \frac{1}{\kappa_j^2} \frac{(K - \xi_j)(\bar{\xi} - \xi_j)}{(K - (1-\pi)\xi_j)^2} > 0 \\ &\Leftrightarrow \quad \sum_{k=1}^m \frac{\xi_k}{\kappa_k^2} \sum_{j=1}^m \frac{1}{\kappa_j^2} \frac{K - \xi_j}{(K - (1-\pi)\xi_j)^2} > \sum_{k=1}^m \frac{1}{\kappa_k^2} \sum_{j=1}^m \frac{\xi_j}{\kappa_j^2} \frac{K - \xi_j}{(K - (1-\pi)\xi_j)^2}. \quad (46)\end{aligned}$$

The jkth terms cancel when $j = k$. For $j \neq k$, collect together the relevant jkth terms, so that a sufficient condition for the above inequality to hold is that, for all $j \neq k$,

$$\begin{aligned}\frac{1}{\kappa_j^2 \kappa_k^2} \left[\frac{\xi_k (K - \xi_j)}{(K - (1-\pi)\xi_j)^2} + \frac{\xi_j (K - \xi_k)}{(K - (1-\pi)\xi_k)^2} \right] &> \frac{1}{\kappa_j^2 \kappa_k^2} \left[\frac{\xi_j (K - \xi_j)}{(K - (1-\pi)\xi_j)^2} + \frac{\xi_k (K - \xi_k)}{(K - (1-\pi)\xi_k)^2} \right] \\ \Leftrightarrow \quad \frac{(\xi_k - \xi_j)(K - \xi_j)}{(K - (1-\pi)\xi_j)^2} &> \frac{(\xi_k - \xi_j)(K - \xi_k)}{(K - (1-\pi)\xi_k)^2}. \quad (47)\end{aligned}$$

Suppose initially that $\xi_k > \xi_j$, so that the first term of the numerator cancels, then this reduces to

$$\frac{K - \xi_j}{(K - (1-\pi)\xi_j)^2} > \frac{K - \xi_k}{(K - (1-\pi)\xi_k)^2} \quad (48)$$

whenever $\xi_k > \xi_j$. In other words, $(K-\xi)/(K-(1-\pi)\xi)^2$ must be decreasing in ξ. Now

$$\frac{d}{d\xi}\frac{K-\xi}{(K-(1-\pi)\xi)^2} = \frac{1}{(K-(1-\pi)\xi)^2}\left(\frac{2(1-\pi)(K-\xi)}{K-(1-\pi)\xi} - 1\right) < 0$$

$$\Leftrightarrow \quad K-(1-\pi)\xi > 2(1-\pi)(K-\xi) \quad \Leftrightarrow \quad \pi > \frac{K-\xi}{2K-\xi}, \tag{49}$$

which is only satisfied for all ξ if $\pi > \frac{1}{2}$ (because $\xi_i < K$ for all $z_i > 0$ as usual). It remains to be shown that the covariance is also decreasing in π when $n \leq 3$. From (41), and multiplying out ρ,

$$\frac{d\,\mathrm{cov}}{d\pi} < 0 \quad \Leftrightarrow \quad \sum\nolimits_{k=1}^{m}\psi_k \sum\nolimits_{j=1}^{m}\frac{d\psi_j}{d\pi}\rho_j < \sum\nolimits_{k=1}^{m}\psi_k\rho_k \sum\nolimits_{j=1}^{m}\frac{d\psi_j}{d\pi}. \tag{50}$$

This inequality is characterised by two products. The jkth terms cancel when $j = k$. For $j \neq k$, collect together the relevant jkth terms, so that a sufficient condition for the inequality is that, for all $j \neq k$,

$$\psi_k\frac{d\psi_j}{d\pi}\rho_j + \psi_j\frac{d\psi_k}{d\pi}\rho_k < \psi_k\rho_k\frac{d\psi_j}{d\pi} + \psi_j\rho_j\frac{d\psi_k}{d\pi} \quad \Leftrightarrow \quad \psi_k\left(\rho_j-\rho_k\right)\frac{d\psi_j}{d\pi} < \psi_j\left(\rho_j-\rho_k\right)\frac{d\psi_k}{d\pi}. \tag{51}$$

Suppose initially that $\xi_k > \xi_j$ so that $\rho_k < \rho_j$, then this last inequality simplifies to $\psi_k \times d\psi_j/d\pi < \psi_j \times d\psi_k/d\pi$. Now recall that $d\psi_i/d\pi > 0$ if and only if $\xi_i > \bar{\xi}$. If $n = 2$, then $\xi_k > \bar{\xi} > \xi_j \Rightarrow d\psi_k/d\pi > 0 > d\psi_j/d\pi$ (as $\bar{\xi}$ is a weighted sum of ξ_k and ξ_j). Therefore, since $\psi_i > 0$ for all i, the required inequality holds for sure (the case $\xi_k < \xi_j$ follows in exactly the same way).

For $n = 3$, note that there are two possibilities: $\xi_1 < \xi_2 < \bar{\xi} < \xi_3$ and $\xi_1 < \bar{\xi} < \xi_2 < \xi_3$. (Ties cause no problems, as may be readily verified.) The latter of these two cases may be dealt with by reference to (51) alone. Recall that $\rho_i = \psi_i\kappa_i^2$ and hence the inequality in (51) may be written

$$\frac{d\,\mathrm{cov}}{d\pi} < 0 \quad \text{if} \quad \rho_k\left(\rho_j-\rho_k\right)\frac{d\rho_j}{d\pi} < \rho_j\left(\rho_j-\rho_k\right)\frac{d\rho_k}{d\pi} \quad \text{for all } j \neq k. \tag{52}$$

Now, $d\rho_j/d\pi > d\rho_k/d\pi$ if $\xi_j > \xi_k > \bar{\xi}$. To confirm this, differentiate an appropriate function $\rho(\xi)$, constructed from (37) in an obvious way, with respect to ξ, giving

$$\frac{d}{d\xi}\frac{d\rho(\xi)}{d\pi} \equiv \frac{d}{d\xi}\frac{\xi(\xi-\bar{\xi})}{(K-(1-\pi)\xi)^2} = \frac{\xi(K-(1-\pi)\bar{\xi})+(\xi-\bar{\xi})K}{(K-(1-\pi)\xi)^3}, \tag{53}$$

which is certainly positive if $\xi > \bar{\xi}$. Therefore each pair of comparisons required in (52) between jkth elements is satisfied: $\xi_3 > \xi_2 > \bar{\xi} > \xi_1 \Rightarrow \rho_3 < \rho_2 < \rho_1$ and $d\rho_3/d\pi > d\rho_2/d\pi > 0 > d\rho_1/d\pi$.

For the former case, $\xi_1 < \xi_2 < \bar{\xi} < \xi_3$, this comparison cannot be done. Instead, again note that $\rho_1 > \rho_2 > \rho_3$ and that $d\psi_1/d\pi < 0$, $d\psi_2/d\pi < 0$, and $d\psi_3/d\pi > 0$. Now, writing out the full expression in (50), and eliminating the common jkth terms with $j = k$ from both sides,

$$\begin{aligned}\frac{d\,\mathrm{cov}}{d\pi} \quad \Leftrightarrow \quad & \psi_1\underbrace{(\rho_2-\rho_1)}_{-\mathrm{ve}}\underbrace{\frac{d\psi_2}{d\pi}}_{-\mathrm{ve}} + \psi_2\underbrace{(\rho_3-\rho_2)}_{-\mathrm{ve}}\underbrace{\frac{d\psi_3}{d\pi}}_{+\mathrm{ve}} + \psi_3\underbrace{(\rho_1-\rho_3)}_{+\mathrm{ve}}\underbrace{\frac{d\psi_1}{d\pi}}_{-\mathrm{ve}} \\ < \quad & \psi_2\underbrace{(\rho_2-\rho_1)}_{-\mathrm{ve}}\underbrace{\frac{d\psi_1}{d\pi}}_{-\mathrm{ve}} + \psi_3\underbrace{(\rho_3-\rho_2)}_{-\mathrm{ve}}\underbrace{\frac{d\psi_2}{d\pi}}_{-\mathrm{ve}} + \psi_1\underbrace{(\rho_1-\rho_3)}_{+\mathrm{ve}}\underbrace{\frac{d\psi_3}{d\pi}}_{+\mathrm{ve}}.\end{aligned} \tag{54}$$

The problem lies in the very first term. The other left-hand-side terms are negative and the right-hand-side terms are positive. Hence it suffices to show that the absolute value of the first left-hand-side termis smaller than that of the last right-hand-side term. Note that ψ_1 is identical (and positive) in both terms. $|\rho_1-\rho_3| = \rho_1-\rho_3 > \rho_1-\rho_2 = |\rho_2-\rho_1|$, so the second element in the right-hand side term exceeds that in the left-hand side term. It remains to be shown that

$$\left|\frac{d\psi_3}{d\pi}\right| = \frac{d\psi_3}{d\pi} > -\frac{d\psi_2}{d\pi} = \left|\frac{d\psi_2}{d\pi}\right| \quad \Leftrightarrow \quad \frac{\xi_3}{\kappa_3^2}\frac{\xi_3 - \bar{\xi}}{(K-(1-\pi)\xi_3)^2} > \frac{\xi_2}{\kappa_2^2}\frac{\bar{\xi} - \xi_2}{(K-(1-\pi)\xi_2)^2}. \tag{55}$$

Since $\xi_3 > \xi_2$, and it is sufficient to show that this holds for $\pi < 1$ (the case of $\pi > \frac{1}{2}$ has already been proved for all n), this latter inequality will hold if

$$\frac{\xi_3 - \bar{\xi}}{\kappa_3^2} > \frac{\bar{\xi} - \xi_2}{\kappa_2^2} \quad \Leftrightarrow \quad \frac{\xi_2}{\kappa_2^2} + \frac{\xi_3}{\kappa_3^2} > \bar{\xi}\left(\frac{1}{\kappa_2^2} + \frac{1}{\kappa_2^2}\right). \tag{56}$$

This inequality holds: $\bar{\xi} = \sum_{i=1}^{3} \xi_i/\kappa_i^2 / \sum_{i=1}^{3} 1/\kappa_i^2$ by definition and any weighted average of the two higher ξ_is will always be larger than the smallest ξ_i, completing the proof for $n = 3$. □

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